DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

23-0009-AQ TEST BOOKLET

MATHEMATICS

PAPER – II

Time Allowed: 3 hours | **Maximum Marks: 300**

INSTRUCTIONS TO CANDIDATES

Read the instructions carefully before answering the questions: -

1. This Test Booklet consists of **24(twenty-four)** pages and has **75 (seventy-five) items (questions).**
2. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS BOOKLET ***DOES NOT*** HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
3. **Please note that it is the candidate's responsibility to fill in the Roll Number and other required details carefully and without any omission or discrepancy at the appropriate places in the OMR Answer Sheet and the Separate Answer Booklet. Any omission/discrepancy will render the OMR Answer Sheet and the Separate Answer Booklet liable for rejection.**
4. Do not write anything else on the OMR Answer Sheet except the required information. Before you proceed to mark in the OMR Answer Sheet, please ensure that you have filled in the required particulars as per given instructions.
5. Use **only Black Ball Point Pen** to fill the OMR Answer Sheet.
6. This Test Booklet is divided into 4 (four) parts – **Part – I, Part – II, Part - III** and **Part – IV.**
7. All three parts are **Compulsory.**
8. **Part-I consists of Multiple Choice-based Questions.** The answers to these questions have to be marked in the **OMR Answer Sheet** provided to you.
9. **Part-II, Part-III and Part-IV consist of Conventional Essay-type Questions.** The answers to these questions have to be written in the separate **Answer Booklet** provided to you.
10. In Part-I, each item (question) comprises of 04 (four) responses (answers). You are required to select the response which you want to mark on the OMR Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose ***ONLY ONE*** response for each item.
11. After you have completed filling in all your responses on the OMR Answer Sheet and the Answer Booklet(s) and the examination has concluded, you should hand over to the Invigilator ***only the OMR Answer Sheet and the Answer Booklet(s)***. You are permitted to take the Test Booklet with you.
12. **Penalty for wrong answers in Multiple Choice-based Questions:**

THERE WIL BE **PENALTY** FOR WRONG ANSWERS MARKED BY A CANDIDATE.

(i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to the question will be deducted as penalty.

(ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to the question.

(iii) If a question is left blank. i.e., no answer is given by the candidate, there will be no penalty for that question.

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PART - I
(Multiple Choice-based Questions)

Instructions for Questions 1 to 50:

- *Choose the correct answers for the following questions.*
- *Each question carries 3 marks.*
- *No Data Books/Tables are allowed; assume the data if required anywhere.*
- *Unless otherwise mentioned, symbols and notations have their usual meaning.*

[3x50=150]

1. Let R be the set of all real numbers and * be the algebraic structure defined by

$$a * b = a^2 + b^2, \text{ for all } a, b \in R.$$

Then * is ________
   (a) associative and commutative
   (b) associative but not commutative
   (c) commutative but not associative
   (d) neither associative nor commutative

2. Let $G = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} : ad \neq bc \text{ where } a, b, c \text{ and } d \text{ are integers } modulo\ 2\right\}$ be the group w.r.t. matrix multiplication;then the order of G is:
   (a) 5
   (b) 6
   (c) 8
   (d) 10

3. The number of generators in a cyclic group of order $8$is equal to:
   (a) 2
   (b) 3
   (c) 4
   (d) 8

4. If G is a cyclic group of order p, where p is a prime integer > 2, then G has ________
   (a) only one generator
   (b) only two distinct generators
   (c) $2p-1$ generators
   (d) $p-1$ generators

5. Let $f: (\mathbb{Z}, +) \to (\mathbb{R} - \{0\}, .)$ be the homomorphism where $f(2) = \frac{1}{3}$. Then $f(6)$ is equal to:
   (a) 1
   (b) 2
   (c) $\frac{1}{9}$
   (d) $\frac{1}{27}$

6. Let $\boldsymbol{H}$ and $\boldsymbol{K}$ be any two subgroups of a group $\boldsymbol{G}$. If $\boldsymbol{H}$ and $\boldsymbol{K}$ have $\mathbf{12}$ and $\mathbf{24}$ elements respectively, then which of the following cannot be the cardinality of the set $\boldsymbol{HK} = \{\boldsymbol{hk} : \boldsymbol{h} \in \boldsymbol{H}, \boldsymbol{k} \in \boldsymbol{K}\}$?
   (a) 36
   (b) 48
   (c) 64
   (d) 96

7. Let $\boldsymbol{S_6}$ be the permutation group on six symbols $\{\mathbf{1,2,3,4,5,6}\}$. Which of the following permutation is not an even permutation?
   (a) (1 3 5 6 2)
   (b) (1 3 4 6) (2 5)
   (c) (2 6 3 4 5 1)
   (d) (1 2 4) (3 6 5)

8. Let $\boldsymbol{M}$ be the ring of all $\mathbf{2 \times 2}$ matrices over the integers. Consider the set $S = \left\{ \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} : a \in Z \right\}$ to be a subring of $\boldsymbol{M}$. Then S is ____________.
   (a) a left ideal but not a right ideal of M
   (b) a right ideal but not a left ideal of M
   (c) a both-sided ideal of M
   (d) neither a left ideal nor a right ideal of M

9. Which of the following statement is **INCORRECT**?
   (a) A commutative ring, with unity element and without zero divisor is a field.
   (b) A commutative division ring is a field.
   (c) Every finite integral domain is a field.
   (d) A field has no zero divisor.

10. The set $\boldsymbol{Z_7} = \{[\mathbf{0}],[\mathbf{1}],[\mathbf{2}],[\mathbf{3}],[\mathbf{4}],[\mathbf{5}],[\mathbf{6}]\}$ forms a field w.r.t. the operations defined as $[\boldsymbol{a}] + [\boldsymbol{b}] = [\boldsymbol{a} + \boldsymbol{b}]\ \boldsymbol{mod}\ \mathbf{7}$ and $[\boldsymbol{a}][\boldsymbol{b}] = [\boldsymbol{ab}]\ \boldsymbol{mod}\ \mathbf{7}$. The inverse of the element $-[\mathbf{4}]$, i.e., $(-[\mathbf{4}])^{-1}$is:
   (a) [2]
   (b) [5]
   (c) [4]
   (d) [3]

11. The units of the integral domain $\boldsymbol{D} = \{\boldsymbol{a} + \boldsymbol{ib} : \boldsymbol{a}, \boldsymbol{b} \in \boldsymbol{Z}\}$ of Gaussian integers are:
   (a) $-1, 1, -i, i$
   (b) $-1, 1$
   (c) $-i, i$
   (d) $1, -i, i$

12. If $< a_n >$ is a sequence, where $a_n = \left[1 + (-1)^n \frac{1}{n}\right]^n$, then:

(a) $\limsup_{n\to\infty} a_n = 1, \ \liminf_{n\to\infty} a_n = \frac{1}{e}$

(b) $\limsup_{n\to\infty} a_n = e^2, \ \liminf_{n\to\infty} a_n = e$

(c) $\limsup_{n\to\infty} a_n = e^2, \ \liminf_{n\to\infty} a_n = \frac{1}{e^2}$

(d) $\limsup_{n\to\infty} a_n = e, \ \liminf_{n\to\infty} a_n = \frac{1}{e}$

13. If $\langle a_n^{1/n} \rangle$ and $\langle b_n^{1/n} \rangle$ are the sequences, where $a_n = \frac{(2n)!}{(n!)^2}$ and $b_n = \frac{n^n}{(n+1)(n+2)\cdots(n+n)}$,

then:

(a) both $\langle a_n^{1/n} \rangle$ and $\langle b_n^{1/n} \rangle$ converge.

(b) $\langle a_n^{1/n} \rangle$ converges but $\langle b_n^{1/n} \rangle$ diverges.

(c) $\langle a_n^{1/n} \rangle$ diverges but $\langle b_n^{1/n} \rangle$ converges.

(d) both $\langle a_n^{1/n} \rangle$ and $\langle b_n^{1/n} \rangle$ diverge.

14. The infinite series $\sum \frac{n!x^n}{n^n}$ ____________

(a) converges, if $x < e$.
(b) converges, if $x > e$.
(c) diverges, if $x < e$.
(d) converges, if $x \geq e$.

15. Which of the following statements is **INCORRECT**?
(a) Every bounded sequence is convergent.
(b) Every Cauchy sequence is bounded.
(c) Every bounded sequence of real numbers has a convergent subsequence.
(d) If a sequence is bounded and has a unique limit point, then it is convergent.

16. Which of the following functions is uniformly continuous on its respective interval?

(a) $f(x)=\frac{1}{}$, for all $x \in]\ 0,\ 1\ [$.

(b) $f(x)=\frac{1}{x^2}$, for all $x \in [a,\ \infty\ [\ , a>0$.

(c) $f(x)=\frac{1}{x^2}$, for all $x \in]\ 0,\ \infty\ [$.

(d) $f(x)=\sin x^2$, for all $x \in [0,\ \infty\ [$.

17. The integral $\int_0^{\pi/2} \frac{x^m}{\sin^n x} dx$ is convergent only if ____________

(a) $n > m + 1$
(b) $n < m + 1$
(c) $m > n + 1$
(d) $m < n + 1$

18. If $f(x) = x^2$, for all $x \in [0, 4]$ and $P = \{0, 1, 2, 3, 4\}$ be a partition of $[0, 4]$, then $L(P, f)$ and $U(P, f)$ are ____ and ____ respectively.
(a) 12, 21
(b) 10, 20
(c) 13, 25
(d) 14, 30

19. Let $\sum_{n=1}^{\infty} u_n(x)$ and $\sum v_n(x)$ be two infinite series of functions, where $u_n(x)=\frac{(\ 1)^{n-1}}{n+x^2}$ and $v_n(x)=nxe^{-nx^2}$, fot all $x \in [0,1]$. Then, whichof the following statements is **TRUE**?

(a) $\sum u_n$ and $\sum v_n$ both converge uniformly on $[0, 1]$.

(b) $\sum u_n$ converges uniformly but $\sum v_n$ converges point wise.

(c) $\sum u_n$ converges point wise but $\sum v_n$ converges uniformly.

(d) $\sum u_n$ and $\sum v_n$ both converge point wise.

20. If $f(x,y)=\begin{cases} \frac{x^3-y^3}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$, then the function **f** is:

(a) not continuous at (0, 0).
(b) continuous at (0, 0) but does not possess partial derivatives at (0, 0).
(c) continuous at (0, 0) and possesses partial derivatives at (0, 0).
(d) differentiable at (0, 0).

21. $\mathbf{f(x)}$ is a function such that $\mathbf{f'(x) = 2ax + b}$ for all $\mathbf{x}$ and has a minimum value at $\mathbf{x = \frac{5}{2}}$, with the condition that $\mathbf{f(0) = 2}$ and $\mathbf{f(1) = 1}$. Then the function $\mathbf{f}$ is:

(a) $\frac{1}{4}(x^2 - 5x + 8)$

(b) $x^2 + \frac{5}{4}x - 8$

(c) $\frac{1}{4}(x^2 - 3x + 8)$

(d) $x^2 - \frac{5}{4}x + 8$

22. Let $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. Then f has __________

(a) a maximum value at the point $(1, 2)$.
(b) neither a maximum nor a minimum value at $(1, 2)$.
(c) neither a maximum nor a minimum value at $(-1, -2)$.
(d) a maximum value at $(-1, -2)$.

23. The derivative of the analytic function $\mathbf{w = f(z)}$, where $\mathbf{f(z) = u(x, y) + i\, v(x, y)}$, with respect to $\mathbf{z}$ in polar form is:

(a) $\frac{dw}{dz} = e^{i\theta} \frac{\partial w}{\partial r}$

(b) $\frac{dw}{dz} = e^{-i\theta} \frac{\partial w}{\partial r}$

(c) $\frac{dw}{dz} = \frac{1}{r} e^{-i\theta} \frac{\partial w}{\partial \theta}$

(d) $\frac{dw}{dz} = -\frac{1}{r} e^{i\theta} \frac{\partial w}{\partial \theta}$

24. The value of the complex integral $\int_C (\bar{z})^2 dz$ over the circle $|z - 1| = 1$, is:

(a) $\frac{3\pi i}{2}$

(b) $2\pi i$

(c) $3\pi i$

(d) $4\pi i$

25. Let $\mathbf{C} : |\mathbf{z}-\mathbf{a}| = \mathbf{r}$ be a circle and $\mathbf{n}$ be a finite positive integer. Then the value of the complex integral $\int_C \frac{dz}{(z-a)^n}$, when $\boldsymbol{n} = \mathbf{1}$ and $\boldsymbol{n} \neq \mathbf{1}$ respectivelyare:

(a) $\boldsymbol{\pi}\mathbf{i}$ and $\mathbf{0}$

(b) $\mathbf{0}$ and $\mathbf{2\pi i}$

(c) $\mathbf{2\pi i}$ and $\mathbf{0}$

(d) $\mathbf{2\pi i}$ and $\mathbf{-2\pi i}$

26. If $\mathbf{C}$ is the square whose vertices are $-\mathbf{i}, \mathbf{2}-\mathbf{i}, \mathbf{2}+\mathbf{i}$ and $\mathbf{i}$, then the value of the complex integral $\int_C \frac{\cos \pi z}{z^2-1}\, dz$ is:

(a) 0

(b) π

(c) $\genfrac{}{}{0pt}{}{1}{2}$

(d) $\genfrac{}{}{0pt}{}{1}{2}$

27. The integral $\int_C \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^3}\, dz;\ C:|z|=1$, is:

(a) $\frac{21}{32}\pi i$

(b) $\frac{21}{16}\pi i$

(c) $\frac{3}{\ }\pi\imath$

(d) $\frac{3}{\ }\pi\imath$

28. The Laurent's expansion of the function $f(z)=\dfrac{z}{(z-1)(z-3)}$ in the region0 < |z- 1| < 2 is:

(a) $\dfrac{1}{2(1-z)} \quad \dfrac{3}{4}\sum_{0}^{\infty}\left(\dfrac{z-1}{2}\right)^{n}$

(b) $\dfrac{1}{2(z-1)}+\dfrac{3}{2}\sum_{0}^{\infty}\left(\dfrac{z-1}{2}\right)^{n}$

(c) $\dfrac{1}{2(1-z)}+\dfrac{3}{4}\sum_{0}^{\infty}\left(\dfrac{z-1}{2}\right)^{n+1}$

(d) $\dfrac{2}{(z-1)}-\dfrac{3}{4}\sum_{0}^{\infty}\left(\dfrac{z-1}{2}\right)^{n}$

29. The function $f(z)=\dfrac{z-\sin z}{z^{5}}$, at z = 0 has ___________

(a) a removable singularity
(b) an isolated essential singularity
(c) a non-isolated essential singularity
(d) a pole of order 2

30. The residue of the function $f(z)=\dfrac{z^{2}-2z}{(z+1)^{2}(z^{2}+4)}$ is:

(a) $\dfrac{-14}{25}$

(b) $\dfrac{-4}{25}$

(c) $\dfrac{4}{25}$

(d) $\dfrac{14}{25}$

31. The value of the integral $\int_{C}\dfrac{f'(z)}{f(z)}dz$, over the circle C : |z| = 3, where $f(z)=\dfrac{(z^{2}+1)^{2}}{(z^{2}+3z+2)^{3}}$, is equal to:

(a) $-2\pi i$
(b) $2\pi i$
(c) $-4\pi i$
(d) $4\pi i$

32. The number of non-degenerate basic feasible solutions of the system $x_1 + x_2 + 2x_3 = 4$, $2x_1 - x_2 + x_3 = 2$ are:
(a) 0
(b) 1
(c) 2
(d) 3

33. If one or more artificial variables appear in the basis at positive level and the optimality criterion is satisfied, then ______________

(a) the problem has an unbounded solution
(b) the optimal solution is degenerate
(c) the problem has no feasible solution
(d) the problem may have an unbounded solution or an infeasible solution

34. Which of the following statements is **NOT CORRECT**?
(a) Replacing $\leq$ sign by $=$ sign in a constraint of a linear programming problem can result in a more restrictive solution space.
(b) In a two-dimensional linear programming problem, the objective function can assume the same value at two distinct extreme points.
(c) In real life situations, the variables in a linear programming model can be unrestricted in sign.
(d) Changes in the coefficients of the objective function of a linear programming problem will definitely result in changing the optimal values of the variables.

35. Which of the following statements **IS CORRECT**?
(a) The primal problem must always be of the maximization form.
(b) If the primal problem has an unbounded optimal solution, then the dual of the same has nofeasible solution.
(c) If the primal problem has an infeasible solution, then its dual always has an unbounded optimum.
(d) The addition of a new constraint improves the objective value of the linear programming problem.

36. Which of the following statements **IS INCORRECT**?
(a) A balanced transportation problem may not have any feasible solution.
(b) The basic requirement for using the transportation technique is that the transportation model is balanced.
(c) The transportation technique essentially uses the same steps of the simplex method in linear programming problem.
(d) The multipliers u_i and v_i in the transportation technique are essentially the dual variables of the linear programming representing the transportation problem.

37. The partial differential equation formed by eliminating the constants **a** and **p** from the relation $\mathbf{z = ae^{pt} \sin px}$ is given by:

(a) $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial t}\right)^2 = 0$

(b) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial t^2} = 0$

(c) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial t^2} = 2$

(d) $t\frac{\partial^2 z}{\partial x^2} + x\frac{\partial^2 z}{\partial t^2} = 0$

38. The particular integral of the differential equation $\mathbf{(4D^2 - 4DD' + D'^2)z = 4\log(x + 2y)}$ is given by:

(a) $x\log(x + 2y)$

(b) $\frac{1}{4}x \log(x+2y)$

(c) $x^2 \log(x+2y)$

(d) $\frac{1}{2}x^2 \log(x+2y)$

39. The complete integral of the partial differential equation $\mathbf{py = 2xy + \log q}$ is:

(a) $z = x^2 - ax + \frac{1}{a}e^{ay} + b$

(b) $z = x^2 - ax + e^{y/a} + b$

(c) $z = x^2 + ax + \frac{1}{a}e^{ay} + b$

(d) $z^2 = x + ax^2 + e^{y/a} + b$

40. The differential equation $x\frac{\partial^2 u}{\partial x^2} + 3\frac{\partial^2 u}{\partial x \partial y} + y\frac{\partial^2 u}{\partial y^2} + 17\frac{\partial u}{\partial x} = 0$ is hyperbolic, if:

(a) $xy < \frac{4}{9}$

(b) $xy < \frac{9}{4}$

(c) $xy < \frac{}{4}$

(d) $xy < 9$

41. Let $u(x,t)$ be the solution of the heat equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}; x\in[0,1], t>0,$ with the conditions $u(x,0)=\sin \pi x; x\in[0,1]$ and $u(0,t)=u(1,t)=0; t>0$. Then $u\left(x,\frac{1}{\pi^2}\right)$ is equal to:

(a) $e \sin \pi x$

(b) $\frac{1}{e}\sin \pi x$

(c) $e \cos \pi x$

(d) $\frac{1}{e}\cos \pi x$

42. Newton-Raphson iteration formula for finding cube root of the number **c (c > 0)** is:

(a) $x_{n+1}=\frac{2x_n^2+c}{3x_n^2}$

(b) $x_{n+1}=\frac{2x_n+c}{3x_n^2}$

(c) $x_{n+1}=\frac{2x_n^3-c}{3x_n^2}$

(d) $x_{n+1}=\frac{2x_n^3+c}{3x_n^2}$

43. The value of the integral $\int_0^4 e^x dx$ by Simpson's rule, if e = 2.718, e^2 = 7.389, e^3 = 20.085, e^4 = 54.598 is equal to:

(a) 53.259
(b) 53.863
(c) 54.347
(d) 54.998

44. If the differential equation $\frac{dy}{dx}=-2xy^2, y(0)=1$ is solved numerically by Euler's method, with step size $h = 0.2$, the approximate value of $y(0.6)$ is:

(a) 0.7846

(b) 0.7885

(c) 0.8745

(d) 0.8954

45. A second-degree polynomial passes through the points $(0,3), (1,6), (2,11), (3,18)$ and $(4,27)$. The polynomial is given by:
   (a) $x^2 + x + 1$
   (b) $x^2 + 3x + 2$
   (c) $x^2 + 2x + 3$
   (d) $x^2 + 6x + 2$

46. The order of convergences in Newton-Raphson's method is:
   (a) 0
   (b) 1
   (c) 2
   (d) 3

47. The decimal equivalent of the octal number **136.54** is:
   (a) 84.1250
   (b) 94.6875
   (c) 97.6875
   (d) 94.1250

48. The complement of the Boolean expression $f(x,y,z) = xyz + xyz' + x'y'z + x'yz$ is:
   (a) $(x + y')(x + z')$
   (b) $(x' + y')(y' + z)$
   (c) $(x' + y')(x + z')$
   (d) $(x' + z)(y' + z')$

49. The Euler's equation of motion in x-direction is given by:
   (a) $\frac{du}{dt} = X + \frac{1}{\rho}\frac{\partial p}{\partial x}$
   (b) $\frac{du}{dt} = X - \frac{1}{\rho}\frac{\partial p}{\partial x}$
   (c) $\frac{\partial u}{\partial t} = X - \frac{1}{\rho}\frac{\partial p}{\partial x}$
   (d) $\frac{\partial u}{\partial t} = X + \frac{1}{\rho}\frac{\partial p}{\partial x}$

50. The image of source $+m$ with respect to a circle is a source $+m$ at the inverse point and ________

(a) a source $+m$ at centre
(b) a source $+m$ at the same point
(c) a sink $-m$ at the centre
(d) a sink $-m$ at the same point

PART - II

Instructions for Questions 51 to 63:

- *Answer any 10 (TEN) out of the thirteen questions.*
- *Each question carries 5 marks.*
- *No Data Books/Tables are allowed; assume the data if required anywhere.*
- *Unless otherwise mentioned, symbols and notations have their usual meaning.*

[5 x 10 = 50]

51. Prove that the orders of the elements a and $x^{-1}ax$are equal. Here,a and x are any two arbitrary elements of a group G.

52. If **H** and **K** are any two finite subgroups of a group **G**, then prove that –

$$O(HK) = \frac{O(H)\cdot O(K)}{O(H\cap K)}.$$

53. Show that the function **f** defined as $f(x)=\begin{cases}\frac{1}{2^n}, \text{ if } \frac{1}{2^{n+1}} < x \le \frac{1}{2^n}, (n=0,1,2,\ldots)\\ 0, \text{ if } x=0\end{cases}$ is integrable on $[0,1]$, although it has an infinite number of points of discontinuity.

54. Show that the volume of the largest rectangular parallelepiped inscribable in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, is $\frac{8abc}{3\sqrt{3}}$.

55. If the complex function $f(z)$ is analytic and $|f'(z)|$ is the product of a function of x and a function of y, then show that $f'(z) = e^{(\alpha z^2+\beta z+\gamma)}$, where α is a real number and β and γ are complex constants.

56. If the complex function $f(z)$ is analytic inside and on a closed curve **C** around the origin, then prove that $\left(\frac{a^n}{n!}\right)^2 = \frac{1}{2\pi i}\int_C \frac{a^n e^{az}}{n!z^{n+1}}dz$. Hence, deduce that,

$$\sum_{n=0}^{\infty}\left(\frac{a^n}{n!}\right)^2 = \frac{1}{2\pi}\int_0^{2\pi} e^{2a\cos\theta}d\theta$$

57. Given below is the unit cost array with supplies a_i; $i = 1, 2, 3$ and demands b_j; $j = 1, 2, 3, 4$

		1	2	3	4	a_i
	1	8	10	7	6	50
Source	2	12	9	4	7	40
	3	9	11	10	8	30
	b_j	25	32	40	23	120

Find the optimal solution to the above Hitchcock problem.

58. Use Cauchy's method of characteristics for solving the partial differential equation $zp + q = 1$, when initial data curve is $x_0 = \mu,\ y_0 = \mu,\ z_0 = \frac{\mu}{2},\ 0 \le \mu \le 1.$

59. Solve the following linear partial differential equation –

$$\frac{\partial^2 z}{\partial x^2} - 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = e^{2x+3y} + \sin(x - 2y).$$

60. Evaluate the integral $\int_0^1 e^{-x^2} dx$ by using the Simpson's one-third rule.

61. In an examination the number of candidates who secured marks between certain limits were as follows:

Marks	9-19	20-39	40-59	60-79	80-99
No. of candidates	41	62	65	50	17

Estimate the number of candidates who secured fewer than 70 marks.

62. Express the Boolean function $f(x, y, z) = (xy' + xz)' + x'$ to its conjunctive normal form.

63. Deduce the principle of energy from the Lagrange's equations of motion.

PART - III

Instructions for Questions 64 to 71:

- *Answer any 5 (FIVE) out of the eight questions.*
- *Each question carries 10 marks.*
- *No Data Books/Tables are allowed; assume the data if required anywhere.*
- *Unless otherwise mentioned, symbols and notations have their usual meaning.*

[10 x 5 = 50]

64. Prove that a subgroup H of a group G is a normal subgroup of G if and only if the product of any two right cosets of H in G is again a right coset of H in G.

65. If R is a commutative ring with unity and $S$is an ideal of R, then prove that the ring of residue classes R/S is an integral domain if and only if S is a prime ideal of R.

66. Prove that a necessary and sufficient condition for a sequence $< x_n >$ to be a convergent sequence is that for any arbitrarily chosen small positive number $\in$, there exists a positive integer m such that, $|x_{n+p} - x_n| < \in$, for all $n \geq m$ and $p > 1$.

67. Prove that the derivative of an analytic function is also analytic.

68. Solve the following linear programming problem:

Max. $Z = x_1 + 2x_2 + 3x_3 - x_4$

s.t.
$$\begin{aligned} x_1 + 2x_2 + 3x_3 \quad &= 15 \\ 2x_1 + x_2 + 5x_3 \quad &= 20 \\ x_1 + 2x_2 + x_3 + x_4 &= 10 \end{aligned}$$

and $x_1, x_2, x_3, x_4 \geq 0$

69. A rod of length l with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly cooled to $0°C$ and are kept at that temperature. Find the temperature function $u(x, t)$.

70. Use Newton-Raphson method to find the approximate value for the real root of the equation $x \log_{10} x - 1.2 = 0$ correct to five decimal places.

71. Show that the moment of inertia of a right circular solid cone, whose height is h and the radius of whose base is a, is $\frac{3Ma^2}{20}\left(\frac{a^2+6h^2}{a^2+h^2}\right)$ about a slant side and $\frac{3M}{80}(4a^2+h^2)$ about a line through the centre of gravity of the cone perpendicular to its axis.

PART - IV

Instructions for Questions 72 to 75:

- *Answer any 2 (TWO) out of the four questions.*
- *Each question carries 25 marks.*
- *No Data Books/Tables are allowed; assume the data if required anywhere.*
- *Unless otherwise mentioned, symbols and notations have their usual meaning.*

[25 x 2 = 50]

72.

(a) If **H** is a normal subgroup of a group **G** and **K** a normal subgroup of **G** such that $\mathbf{H} \subset \mathbf{K}$, then prove that the quotient group $\frac{\mathbf{G}}{\mathbf{K}}$ is isomorphic to $\frac{G/H}{K/H}$.

(b) Prove that the equation $\boldsymbol{z^4 + z^3 + 1 = 0}$ in the complex variable $\boldsymbol{z}$ has exactly one root in the first quadrant.

73.

(a) If a function $\boldsymbol{f}$ is continuous on a closed interval $[\boldsymbol{a}, \boldsymbol{b}]$ and if $\boldsymbol{f(a)}$and $\boldsymbol{f(b)}$are of opposite signs, then prove that there exists atleast one point $\boldsymbol{c} \in]\boldsymbol{a}, \boldsymbol{b}[$such that $\boldsymbol{f(c) = 0}$.

(b) Show that the function $f(x,y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}} & ,(x,y) \neq (0,0) \\ 0 & ,(x,y) = (0,0) \end{cases}$ is continuous at (0, 0), possesses partial derivatives but is not differentiable at (0, 0).

74.

(a) The only singularities of a single valued complex function f(z) are poles of order 2 and 1 at the points z = 1 and z = 2, with residues of these poles 1 and 3 respectively.If $f(0) = \frac{3}{2}$ and $f(-1) = 1$, determine the function f(z).

(b) Solve the dual of the following linear programming problem by simplex method:

Max. $Z = 2x_1 + x_2$

s.t. $x_1 + 4x_2 \leq 24$

$x_1 + 2x_2 \leq 14$

$2x_1 - x_2 \leq 8$

$x_1 - x_2 \leq 3$

and $x_1, x_2 \geq 0$

75.

(a) Solve the ordinary differential equation $\frac{dy}{dx} = x + y$, with initial condition $y(0) = 1$ by Runge-Kutta method (of order four), from $x = 0$ to $x = 0.4$ taking $h = 0.1$.

(b) If $\boldsymbol{u\,dx + v\,dy + w\,dz = d\theta + \lambda\,d\psi}$, where $\boldsymbol{\theta, \lambda, \psi}$ are the functions of $\boldsymbol{x, y, z, t}$, prove that the vortex lines at any time are the lines of intersection of the surface $\boldsymbol{\lambda = constant}$ and $\boldsymbol{\psi = constant}$.

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